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January 2015

Perspective: of Time and Eternity

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Perspective: of Time and Eternity

Cover Page Footnote

The piece that I have written is about the transition, in Italian religious painting of the 14th and 15th centuries, from perspective systems assuming movement (under certain restrictions) to the perspective of the fixed point. It concerns art history and mathematics. My hope is that among the readership of The Journal of Humanistic Mathematics are some who would be interested in its interdisciplinary theme. I must confess that my level of mathematical competence is very low, and the mathematical content of my piece may be risibly naive. Nevertheless, I am hopeful that the piece is interesting and that the mathematical suppositions of religious picture-making (specifically, the altarpiece), shift in tandem with changing piety---that, in other words, mathematics, perception, dogmatics and empathy are intimately connected.

Perspective: of Time and Eternity

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Abstract

This paper considers geometric perspective in relation to devotional requirements in Italian religious painting from about 1250 to about 1450. The content of the altarpiece consisted in antithetical elements—the graphic exposition of Christian dogmatics, and a *dramatis personae* increasingly to be identified in empathetic terms. The one-point perspective system that was invented towards the end of that period, then, presented an opportunity and a difficulty. It enabled the creation of a naturalistic space, aiding empathetic identification with psychologically plausible individuals in the pictured world. On the other hand, whilst superficially the space marked out by the geometry of the vanishing point coincided with that of Christian hierarchy, it threw dogmatics into crisis, as it set picture space against picture plane. In addition to a humanistic imperative, the method was driven by a mathematical one, consideration of which allows us to see it as the last stage in a process of simplification of the conditioning geometry governing representation within the altarpiece. The paper shows that, despite first impressions, the earlier mathematical perspective systems were systematic. It demonstrates that, as there was a stand-off between dogma and empathy, there was one between iconic stability and mobile perspective in the earlier perspective methods, and one between static viewpoint and imaginative mobility in the invention. From within these antitheses came the necessity for very devious negotiations of the timeless and the historical—all compromising one another in one way or another—and, after the invention of the one-point system, the proposition of an expanded mental life both in those depicted and in those observing.

Keywords: *perspective, viewpoint, altarpiece, architecture, Renaissance, Alberti, Brunelleschi.*

Christian ritual is exercised by the need to establish hierarchy. The altarpiece, which is the specific locus of painting in the present discussion, participates in the task of organizing figurative elements in arrangements that make clear the dogmas of the religion. And the schemata of such exegesis are essentially diagrammatic and planar. As the illusion of volume or picture space was co-opted ever more purposively in the Early Modern Period to add a layer of physical exemplification to the timeless verity of dogma, the schemata had to be retained, but the relationship of plane and plasticity became problematic, and increasingly so. With the invention of the one-point perspective system—one committed unambiguously to the creation of picture space—the geometry of the vanishing point and that by which hierarchy was stated could be made to coincide. But the space created by the one and the plane of the other were in a conflict that forced anomaly upon naturalism and insinuated qualification within dogma. The one-point perspective system was, at the same time, the culmination of a process of development of mathematical perspective towards the stability of the rendered scene and a crisis for the verity intended to underlie it.

Leon Battista Alberti (1404-72) wrote his treatise *On Painting* (*De pictura*) in 1435. With its appearance in Italian (*Della pittura*) in the following year, he addressed both the scholarly and the practical reader or painter.¹ Included in his instruction was advice about the mathematics of painting; specifically how to apply the one-point perspective method to the picture plane and create a mathematically-determined picture space. By following a sequence of geometrical instructions, the painter produces a perspective paving grid that serves as a ruler by which to establish the sizes of things according to their distance in picture space from the picture plane.²

What was, on the picture plane, a trapezium was, in picture space, a square (see Figure 1). The configuration of the whole pavement was fixed according to a point at some determinate position distant from the picture plane. This point was treated as the ideal viewing location. It is easy for us to see that a trapezium, if thought to be the perspective representation

¹There is some disagreement about the order in which the Latin and Italian versions of the treatise appeared. Rocco Sinisgalli [5, pages 3–14] has recently argued the priority of the Italian.

²See [1, 2]. Alberti's perspective method is discussed in detail by Martin Kemp in [4, pages 21–24].

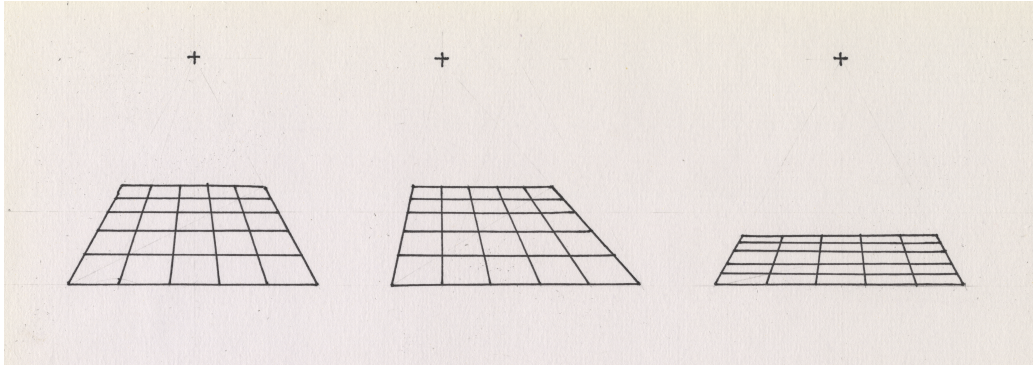


Figure 1: Pavements.

of a square, carries this construction-, projection- or view-point (viewpoint) within its geometry. With the height of the point, the angles of the trapezium change: with the inequality of the angles to left and right, the point is to be thought shifted sideways. Its distance from the construction-point changes with the distance between the near and distant transversals (see Figure 2).

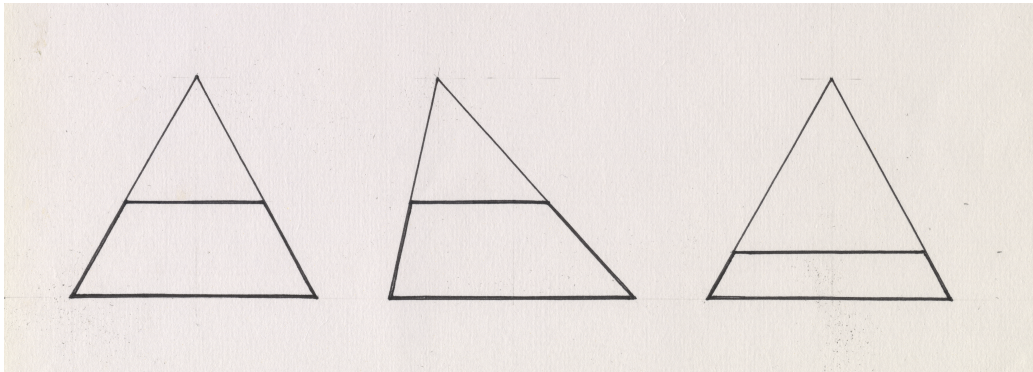


Figure 2: Trapezia.

The one-point perspective system that Alberti described—the *Costruzione Legittima*—makes clear that the size of depicted objects is a function of their distance from the construction-, projection- or viewpoint (see Figure 3). But more than that, the fixity of that point is what gives rationality to the whole measuring process that is implicit in the perspective pavement. Created according to the rule of the fixed point, this painted world is itself supremely rational.

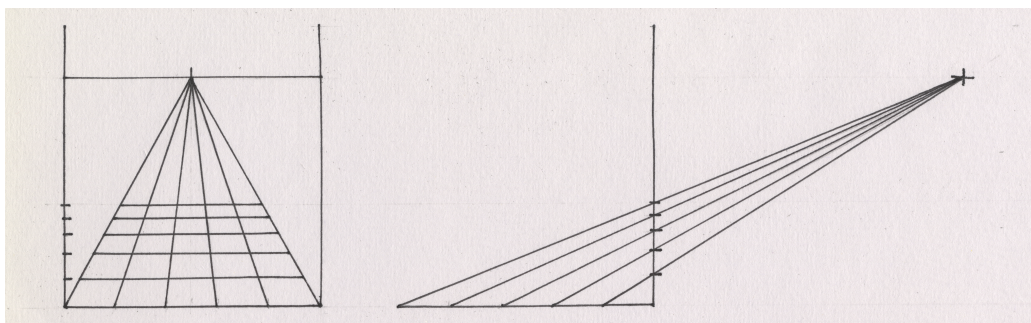


Figure 3: The one-point perspective system: The *Costruzione Legittima*, after Alberti, *De pictura/Della pittura*.

When it is recognized that the rationalism is the product of the notion of point of view (or, better, point of construction or point of projection), the importance for the invention of this aid for painters of an earlier perspective demonstration becomes evident.

Filippo Brunelleschi (1377-1446), the builder of the dome of Florence Cathedral (1420-36), was so celebrated that he was honoured by a written biography, an accolade normally reserved for princes, prelates and generals. His biographer, Antonio Manetti (1423-97), was writing probably forty years after Brunelleschi's death; but he had known the old man. And a passage in which Manetti describes Brunelleschi's perspective demonstration is eminently lucid. As Manetti said, he had had the piece in his own hands many times.³ Recognising the essentially scientific and rationalistic nature of the demonstration, he wrote as follows:

...he [Brunelleschi] propounded and realized what painters to-day call perspective, since it forms part of that science which, in effect, consists in setting down properly and rationally the reductions and enlargements of ... objects as perceived by the eye of man: ... he originated the rule that is essential to whatever has been accomplished since his time in that area. We do not know whether, centuries ago, the ancient painters ... knew about perspective or employed it rationally. If indeed they did employ it

³“E io lo avuto in mano e ueduto piu uolte a mia dj e possone rendere testimonianza.” in [6, page 45, lines 202–03].

by rule (I did not previously call it a science without reason) . . . whoever could have taught it to him had been dead for centuries and no written records about it have been discovered, or if they have been, they have not been understood. But he with his industry and intelligence—either the one—rediscovered it or—the other—invented it.⁴

Manetti proceeds to describe the panel, about one foot square, upon which Brunelleschi painted the Florence Baptistry, seen as it were from a position within the central doorway of the Cathedral. He silvered and burnished the sky. Then he drilled a hole in the panel at the point on the painted Baptistry door where the direct and horizontal line of sight of the observer, standing in the Cathedral doorway, would strike the real Baptistry door.⁵

In his narrative, Manetti omits to say that he took the panel with him to the Cathedral doorway. Whoever would have done so would have been able to grasp the main purpose of Brunelleschi's demonstration without difficulty.

⁴This is from page 42 of [6]. I have modified the translation on from 'whoever'. The whole passage is as follows: 'Cosj ancora in que tenpi e misse et innatto luj proprio quello che dipintorj oggi dicono prospettiva, perche ella e una parte di quella scienza che e in effectto porre bene e con ragione le diminuizionj e acrescimenti, che appaiono agli occhi degli huomini delle cose di lunghi e d'apresso, casamenti, piani e montagnie e paesi d'ognj ragione e innognj luogo le figure e l'altre cose di quella misura che s'appartiene a quella distanza che le si mostrano di lunghi. E da lij e nato la regola, che e la inportanza di tutto quello che di cio se fatto da quel tenpo in qua. Ed e piu forte, che non si sa, se que dipintorj antichi di centinaia d'annj indietro, che si crede, che fussono buonj maestrj, al tenpo di buoni scultori, se lo sapeuano e se lo feciono con ragione. Ma se pure lo fecione con regola, che senza cagione non dico io scienza poco di sopra, come fece poi luj, chi lo potesse insegnare alluj, era morto di centinaia d'anni, e iscritto non si truoua, e se si truoua, non e inteso. Ma la sua industria e sottiglieza o elle ala ritruouo o ella ne fu inventrice.' [lines 143–157].

⁵Brunelleschi made another perspective demonstration panel, a view of the Piazza della Signoria facing the southwest corner of the Palazzo Vecchio. Manetti describes it more briefly, in [6, pages 44–47, lines 204–227]. It seems to have been a demonstration of the object set obliquely rather than frontally to the picture plane. The construction was familiar in narrative painting by the time. It could be used more or less emphatically. In softest form it appears, for example, in Giotto's *Feast of Herod* in the Peruzzi Chapel in Santa Croce. Closer to the time of Brunelleschi's demonstrations, Lorenzo Monaco used it ostentatiously—this time in an altarpiece with a narrative subject—in the *Adoration of the Magi* of c. 1422 in the Uffizi. The construction lends itself to indicating the three-dimensionality of solid objects. It is less effective for delineating spacial voids.

He would have stood looking at the Baptistry through the hole in the panel, the painting on the panel turned to face the Baptistry. He would then have held up a mirror between the Baptistry and the panel, thus obliterating the view by obstructing it with the other view, the painted version of it in reflection! At this point, the observer would readily have taken up the challenge of the panel: if he could get the correct configuration of viewing position and distance of mirror, he would be able to match up the real and the pictured Baptistries perfectly. According to Manetti, the painting was executed with the precision of a miniaturist. It would be congenial to think that the observer, moving the mirror into and out of the line of sight, would have been unable to tell the one from the other. On a breezy day, the reflected clouds would take the panel beyond painting to an “animated” moving illusion. But, even if the difference between the two was to be discerned, the point was clear. The building and the rendering of it could be substituted only so long as the viewing distance and the viewing position were absolutely right. There could be no departure from that point in space.

Brunelleschi showed that the painted and the real could be perceptually identical, and he demonstrated the necessary mathematical conditions. The date of the demonstration is debated by students of the work of Brunelleschi.⁶

⁶See, for example, [3]; a letter of Domenico da Prato of 1413 refers to Brunelleschi as an excellent ‘prospettivo’ [page 330]. However, it is not necessary to insist that, by this time, he had developed the means to create the demonstration panel. The first example of the method given over to painting is Masaccio’s *Trinity Fresco* of 1427. Brunelleschi was almost certainly Masaccio’s architectural and perspectival mentor. An earlier example of perspective by a friend of Brunelleschi, the relief of *St. George and the Dragon* below the statue of *St. George* for Orsanmichele, of 1417, by Donatello, demonstrates a less mathematically rigorous perspective. If Brunelleschi assisted Masaccio or used him as his perspectival cat’s paw, more than fourteen years between the invention of the method in the Baptistry panel—already almost ready for use by painters—and its demonstration in public painting seems like a long time. This is not the place to discuss Brunelleschi’s Baptistry demonstration comprehensively. How it was produced and the practical use to which the lesson derived from it was to be put are questions to pursue elsewhere. Here, emphasis is on its core demonstrative point, which is that a picture will exactly match a building only from a projection *point*—something that a surveyor would readily grasp. The significance of the geometric point was made clear. In addition, the use of the mirror—because it was controlled by the observer—drew attention to the point’s crucial viewing-distance coordinate. That is, the ratio of the distance from the eye to the mirror and back to the painted Baptistry to the distance to the real building was equal to the ratio of the size of the painted to the real building.

But it is put most plausibly after 1420; and the occasion of its making was most likely in connection with the dome of Florence Cathedral and the need to regulate the growth of a structure that could not have a conventional formwork to follow. If the painted Baptistry were the same as the built Baptistry, the order of events was reversible. A painted Baptistry (or Dome) could anticipate the form of a built one.

This identity of the real with the painted depended upon the geometry of similar triangles. A triangle subtended at the real thing could be identical, except in size, to one subtended at a picture plane. In turn, that geometry—the geometry of the surveyor—depended upon a geometrical conception of vision: light travels in straight lines.⁷ The one-point perspective system acquired enormous authority in painting. Its power was, if anything, confirmed later by the fact that the camera works on the same basic geometry.

Brunelleschi's demonstration panel does not survive. However, a picture that, famously and with spectacular effectiveness, shows off the supreme rationalism and world-making potentiality of the system seems to be a witty homage to it. The anonymous *Ideal City View* in the Museo Nazionale delle Marche in Urbino (see Figure 4) also shows a baptistry in a city square. A city of ideal order is before us: the view of the Florence Baptistry from the Cathedral doorway was a more bustling and less-ordered scene. In place of the octagonal building in Florence is, in the imaginable world in Urbino, a round building with an applied peristyle of classical columns. Identifying the building as a baptistry, the observer could have conceived the cathedral, its partner-building, behind him. If he would have followed the logic of the suggestion, he would conceive Florence Cathedral 'done over' into a classicizing form. There would be a vast dome at least as wide as this, set twice as high; it would be hemispherical. St Peter's in Rome was anticipated in this imaginary church.

⁷The fact that the eye is not itself a point means that, whilst vision might be used by surveying, it is not to be identified with it. Alberti made clear that he was supplying to painters a tool of pictorial construction, not a means of replicating the sensation of vision when he wrote that he would disregard the physiology and optics of the eye. He states the point in a passage interpolated in the Latin version or removed from the Italian: 'Neque hoc loco disputandum est utrum in ipsa iunctura interioris nervi visus, ut aiunt, quiescat, an in superficie oculi quasi in speculo animato imagines figerent. Sed nec omnia quidem oculorum ad visendum hoc munera referenda sunt. Satis enim erit his commentariis succincte quae ad rem pernecessaria sint demonstrasse' ([1, page 19 paragraph 6 line 12], [2, page 47]).



Figure 4: Anon, *Ideal City View*, c.1470, Museo Nazionale delle Marche, Urbino. Formerly attributed to Piero della Francesca [Public domain], via Wikimedia Commons (http://en.wikipedia.org/wiki/Ideal_city accessed December 23, 2014).

The piazza is laid out, it seems, with unfailing rigour. The projection-point or construction-point has absolute authority over the configuration of geometrical shapes on the plane of the panel—buildings and pavements in the picture space. Yet there is a surprising consequence. The observer can, in imagination, quit his/her viewing point and find himself wandering the space, confident that the unexpected will not happen either visually or physically. The space is ordered and the architecture is sound.

However, so long as the observer commits himself to the stationary point of view and disregards this possibility of locomotion in imagination, as Brunelleschi's demonstration panel required, stillness pervades the scene. It did so, in a way, before the Florence Baptistry, with the panel and the mirror. There was a continual process of verification: the mirror was moved back and forth. Establishing the stable and static nature of the Baptistry was the aim of the exercise. And the picture on the panel became the corollary of a blink. With a stationary point of view, we open our eyes again and do not expect the scene to have altered. Any change obliges us to abandon our fixed gaze and pinpoint the new detail using our darting vision.

Therefore, the rationality of the one-point system, the coherence of the depicted world and the unchanging are bound together in the process and the experience of the picture. These qualities are so insistent that we are much inclined to find them wanting in pre-one-point perspective constructions, to the extent of thinking them the product of disordered thinking. Lewis Carroll conjured up for Alice a world no less sudden, changeable and disorientating than, say, *Entry into Jerusalem* of Duccio on the rear of the Siena Cathedral *Maestà* (1308–11), in the Museo dell'Opera del Duomo (see Figure 5).



Figure 5: Duccio di Buoninsegna (c. 1255–1319), *Entry into Jerusalem*, on the rear of the *Maestà*, 1308–11, Tempera on wood, 100×57 cm, Museo dell’Opera del Duomo, Siena. Image from Web Gallery of Art, at http://www.wga.hu/html_m/d/duccio/maesta/verso_1/verso01.html, accessed on December 23, 2014.

Those earlier geometric perspectives, however, are not as unsystematic as they can at first seem when compared with the one-point construction. In order for their rationality to emerge, it is necessary to recognize the very tight connection between rationality and immobility. The one-point system, insisting upon total immobility of projection-point, is totally reconstructible in the rational terms of the architect—plan and elevation. Allow for mobility of viewpoint, and the view ceases to contain, encoded in perspective, all the

measured data required for its construction in reality.⁸ It offers instead a less constrained experience for the eye (if not the mind and imagination).

Geometric perspectives using rectangles and parallelograms and uncoordinated trapezia were used by so-called ‘Primitive’ Italian painters in the Ducento and Trecento. The geometric shapes that configure them can be shown to carry implications of viewing position and at the same time to allow movement for the viewer—within certain limits.

Duccio has been used above as the exemplar of a painter creating chaotic worlds. That is unjust. The challenges of narrative painting, in predellas and frescoes, demanded improvisations and consequently illogicalities that devotional painting—the altarpiece—could largely avoid. The Madonna of the *Maestà*, on the other side of the altarpiece from the *Entry into Jerusalem* sits on a throne whose construction is highly systematic (see Figure 6). It is inlaid with rectangular panels of marble that the painter draws on the picture plane as parallelograms. Clearly, the specification of the viewing point in its three coordinates is absent here. But the painter is not indifferent to the whereabouts of the viewer. Both flanks of the throne being the same, the viewer is assumed to be on axis. There is also a general assumption about viewing level: it is possible to see the horizontal surface of the seat of the throne but not the tops of the tower-like lugs of the back. The inlaid marble panels back up this acknowledgement of viewing height. There are even trapezia on either side; but they are not drawn with the drill-sergeant’s demand that the viewer’s eye-level be absolutely fixed. The left-right symmetry in this case gives a vertical plane of vision. It is cut across by a less precise horizontal zone of vision.⁹

⁸A deficiency, if it may be called that, of the older perspectives is that they supplied insufficient data for the builder. In this observation is a possible answer to Marvin Trachtenberg’s question [8] why the freedom of Trecento spacial drafting—that ‘set of pictorial conventions that would produce the illusion of space, and of the solids set in that space, in an economical and effective manner’ [page 167]—submitted to the restrictiveness of the one-point method [pages 175–76]. It could be that the one-point system exposed the spaces created using the earlier conventions as architecturally ramshackle.

⁹John White in [10], organising his observations largely in Vasarian terms of artistic biography and chronology, offers a systematic account of the various combinations of geometric figures by which painters indicated space in pictures, before one-point perspective was arrived at; see in particular the figure on page 27. Here, the approach is more abstract and considers single quadrilaterals—noting that they carry *a priori* spacial implications—rather than their artful combinations on actual picture planes. Priority is



Figure 6: Duccio di Buoninsegna (c. 1255–1319), *Maestà* (detail, Madonna with Angels and Saints), 1308–11, Tempera on wood, 214 × 412 cm, Museo dell’Opera del Duomo, Siena. Image from Web Gallery of Art, at http://www.wga.hu/html/d/duccio/maesta/maest_01.html, accessed on December 23, 2014.

The parallelogram, when it is taken as the representation of a rectangle in perspective, indicates the side from which it is viewed and, in tandem with other data in the picture, whether it is seen from above or below. The point is made very easily by considering Duccio’s *Rucellai Madonna* (1285) in the Uffizi (see Figure 7).

What is to be emphasised is that a parallelogram is a true representation of a rectangle. Evidently, Duccio intended that the parallelograms on the flank of the throne have the same 90°-angles in picture space as have the rectangles of the face of the throne, parallel to the picture plane.

What restrictions upon viewing position do not apply in such a ‘primitive’ perspective as this? Perhaps a convenient way to begin to answer the question is by conceiving the parallelogram as the shadow of a square. The light is to be striking the projection plane at right angles. If the light had a point

given to determinants of projection rather than the projected surface and the implication of picture space themselves. In other words, the various quadrilaterals on the picture plane, representing rectangles in the picture space, are read as subsequent to the geometric specification of viewing position, in terms of space, plane. line and, at last, point.



Figure 7: Duccio di Buoninsegna (c. 1255–1319), *Rucellai Madonna*, 1285, Tempera on wood, 429 × 290 cm, Galleria degli Uffizi, Florence. Image from Web Gallery of Art, at <http://www.wga.hu/html/d/duccio/rucellai/3ruccela.html>, accessed on December 23, 2014.

source, the square would cast a trapezoidal or irregular-quadrilateral shadow (or, if striking the square at right angles to the plane and exactly in the centre, a square). In the case of the parallelogram, it is necessary to think of the light as coming from a very distant source—even infinitely distant—so that the rays are parallel to one another and not diverging. The square has been set skew to the line of light and the projection plane. In a similar and simpler way, a modern map can be thought of as what the mapped area is like for a viewpoint that can be anywhere within an infinitely deep box of space rising at right angles from the plane and confined by its edges. The parallelogram is a more constrained projection. The eye is not allowed to ride any one of those rays. It is located upon the rays in line along the edge of the block of light that touches the top point or the bottom point (depending upon whether the parallelogram is to be conceived as viewed

from above or below). The block of light has confines on three sides. The fourth, undelimited side is the one from which the viewer looks and on the side of the parallelogram that is being considered the more distant. In other words, the picture is conceived in terms of a viewing plane that stops where the near side of the figure coincides with the projection plane. If this is difficult to conceive, another way to think of it is as follows. If we draw a parallelogram twice and indicate one as having a front and top edge, and the other, a front and bottom edge, the two perspectives flip. That is, there is no intermediary perspective. If there is a perspective void in the middle, each figure has a viewing plane. That plane is of indeterminate length, passing away from the forward vertical edge (see Figure 8).

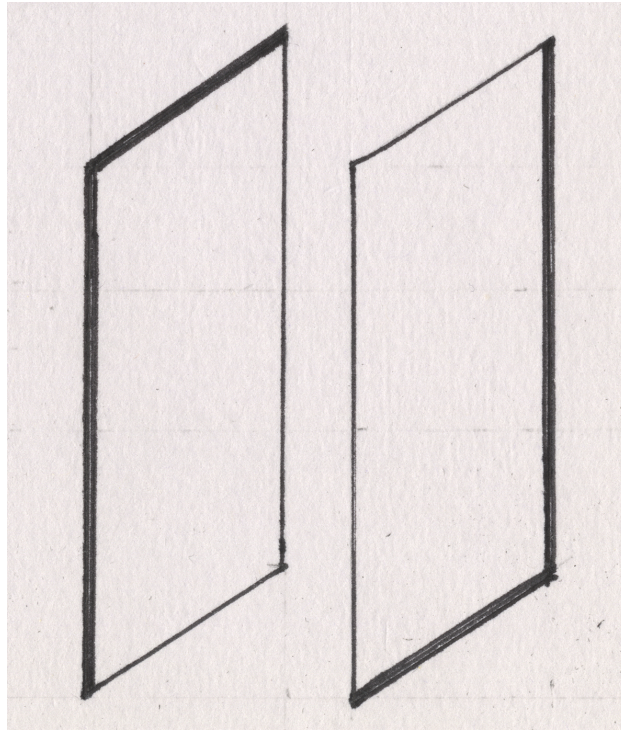


Figure 8: Parallelograms.

Although the front plane of Duccio's throne is parallel with the picture plane and, since it is constructed of rectangles, has an infinite number of viewing points at any distance from it, the flank of the throne is consistent with a viewing position to the left. It is at a height above the level of the arm of the throne. The viewer readily conceives of himself as being to the

side. With the Madonna he has his first encounter, and with the Child who blesses, his second. The intercessory role of the Madonna becomes party to the perspective. Christ's gaze is level; and the devout viewer will suppose himself at that height.

The viewer's freedom to locate himself in space before the picture is considerable. And with it come collisions of spacial logic. Before the one-point system was invented and those anomalies were removed, there is clear evidence that their minimisation was desirable and the pictured scene was to approach closer to stable visual experience.

A more advanced stage in geometrical perspective—or a more restrictive one—is to be found in the work of Giotto (1266/7–1337). Since the *Rucellai Madonna* has served to explain Duccio's perspective, the same iconographic subject can be used for Giotto. His *Ognissanti Madonna* (c. 1310) in the Uffizi calculates the viewer's position with remarkable fastidiousness (see Figure 9 on the next page). It is based principally on the perspective of the trapezium, though parallelograms also appear. The throne stands as-it-were in an axial position, like in the Siena *Maestà*.

Close scrutiny of the picture reveals, however, that the vertical axial line or plane is not as unambiguously placed as it first appears. While the throne is symmetrically placed in the field, the viewer—as was the case for the *Rucellai Madonna*—is off-centre. The difference between Duccio and Giotto is that the latter is supremely conscious of the counterpoint between the architecture of the scene and the position of the viewer. Certainly, that displacement of the axially of the gable of the throne and the Madonna's halo, placed asymmetrically, needs to be recognised as dramatic. In view of the hieratic norms of this kind of religious picture, this shifting of the Madonna's halo and her authority from off the axis of the panel could seem like a demotion. But a kind of differential psychological focus emerges to account for the contrast. While the Madonna's left pupil lies on the axis of the throne, her right eye lies on the vertical diameter of the halo. Both eyes fix the viewer.¹⁰ We find ourselves the object of her gaze and thence her attention. Giotto opens up a world of psychological connection between our place and the pictured.

¹⁰Giotto makes a similar displacement of the viewing line on the throne of the Sultan in the Bardi Chapel. It is subtly to right of axis.



Figure 9: Giotto di Bondone (1267-1337), *Ognissanti Madonna*, c. 1310, Uffizi, Florence. Image [Public domain], via Wikimedia Commons (http://en.wikipedia.org/wiki/Ognissanti_Madonna, accessed December 23, 2014).

The realization that the viewer is to line himself up with the vertical axis of the Madonna's halo, rather than that of the throne and her left eye, is confirmed most emphatically at the bottom of the picture. There, the step that breaks forward on the level of the kneeling angels is depicted asymmetrically, for a vertical viewing plane to the left of the axis of the panel. The parallelograms indicating the obtruding sides of the step are of unequal width, and the central lozenge of the next step is also displaced towards the right. Such displacements are to be accounted for by the viewer conceiving himself located to the left of the vertical axis of the panel. Moving upwards, it becomes apparent that the flanks of the throne are not absolutely identical, but are unequal on the picture surface. The flank on the left is noticeably

more foreshortened in the picture space. In fact, it is narrower on the picture surface by the width of the flat band that runs up the sides of the Madonna's gabled recess. Since the forward piers of the throne are placed equidistantly from the edges of the panel, it must be the case that the recess is not on the axis of the panel. This is confirmed by the observation that the decorative soffit is a broader band at the right springing than at the left. These are small differences but significant for the observer's address to the figures in the picture.

The asymmetry that governs the depiction of the Madonna and Child and the throne does not extend to the placing of the majority of haloes of the attendant saints and angels. However, the remarkable precision and tenacity of Giotto's sense of asymmetrical perspective is revealed in his treatment of the heads within the haloes. Not only does he turn the heads to different degrees from top to bottom on each side, to locate the Madonna and Child as focus of attention within the picture space, but he also differentiates the angle-of-turn of the heads on the left from that of those on the right. This may be seen most clearly in the back row. The heads on the left tend to the profile whilst those on the right tend more to the three-quarter view. Giotto makes minute calculations of angle. For example, the nose of the angel on the left breaks the line of his cheek whereas this does not happen correspondingly on the other side. Such a differentiation is not the result of naturalism consisting in the abandonment of uniformity. It is entirely and minutely consistent with a vertical viewing plane to the left of the axis of the panel. Giotto's conception of viewing angle extends to the idea that the haloes of the foremost standing angels would obscure different amounts of the figures standing behind to right and left. More can be seen of the eye of the saint to the left. At length, it becomes clear that the plane of vision finds no confirmation in the structure of the picture, where everything is oblique and asymmetrical. It seems that here vision and pictorial structure do not conspire to maintain a system of order. The result is something of the nature of a world beyond the grip of Art. Set adrift from the axuality of the iconic object and of the ritual that it implies, the viewer finds himself fixed by the right eye of the Madonna. The coincidence amounts to an acknowledgement, an intercourse between a fictive life and a real one; and disbelief in the fictive is suspended.

The rigorousness of Giotto's adherence to the discipline of his perspective is remarkable. It is a feat of memory as much as geometric neatness that

he should have sustained his rule. Giotto considered each quadrilateral that he drew as implying a general viewing position; he remembered his position throughout the process of designing the picture. He maintained an unremitting commitment, in other words, to the contemplation of the subject as a *scene*.

Giotto has worked very fastidiously upon the surface of the panel for the sake of making as coherent as possible his conception of the space of the picture. The hieratic position of the Madonna should be on axis and on the surface of the altarpiece. Giotto uses all means to shift her from that place—off axis and into picture space. Combined with his very finely-tuned perspective devices is the brute but brilliant logic of setting two angels closer than she to the picture plane, giving them mass by light and shade, and gravity by the convincingness of the pavement upon which they kneel. The Madonna is now spacially remote, as are all things that the eye can know.

The painter exercised an unusual degree of discipline over the observer; the vertical viewing plane is very precisely indicated, and a horizon is established by the moldings halfway up the pillars of the throne. Where these planes intersect is the point at which the viewing line can be thought to strike. But how long the viewing line is—how long the viewing distance—is still uncertain. Indicative of the uncertainty of Giotto's sense of viewing distance is the fact that he included parallelograms (which have no viewing distance) for the angled step in the foreground. There remained, then, the problem of specifying a determinate remoteness and immobility for the viewer.

In his commission of 1426 for the altarpiece of the Carmelites in Pisa, Masaccio (1401–28) followed Giotto, either directly or through an intermediary image, in displacing the Madonna off-axis and setting her securely in picture space, behind foreground angels (see Figure 10 on the next page). He too is very alert to the whereabouts of the observer. It is now possible to trace orthogonals to a single point, and the viewer suspects that a definitive viewing distance accounts for the trapezia that could be scribed at the top and middle levels of the columniated throne.

But what most alerts the viewer to a sense of himself as a rational observer whose interpretative powers are called upon by the painter to make sense in the debate between picture surface and picture space is the treatment of the lutes. One ovoid shape is vertical and the other more or less



Figure 10: Masaccio (1401–28), *Madonna and Child*, from the Pisa Polyptych, 1426, National Gallery, London. Image [Public domain], via Wikimedia Commons ([http://en.wikipedia.org/wiki/Madonna_and_Child_\(Masaccio\)](http://en.wikipedia.org/wiki/Madonna_and_Child_(Masaccio))), accessed December 23, 2014).

horizontal. Their ratios of long-to-short axes are different. And one is yellow and the other brown. Two such unlike things. Yet they are identified as lutes. It is the viewer who brings to the experience his knowledge of space and light—objective phenomena—to account for the differences. So, the consistent behaviour of light performs a sort of geometrical explanation, intersecting and confirming the geometry of vision that operates on exactly the same principles. The viewer construes that the lutes are differently orientated towards the picture plane and therefore the vanishing point, and therefore the viewing line.

Viewpoint—the final viewing condition in the series—is, as we said above, implicit in any trapezium conceived as a square viewed in picture space at right angles to the picture plane. In the *Pisa Madonna and Child*, the

trapezia of the throne are consistent with a single viewpoint (construction- or projection-point).¹¹

The geometrics of Masaccio's picture are all in place. Light, space and viewpoint combine, as they had in Brunelleschi's demonstration panel, in a view arguing a necessary connection between the natural and the rational. However, the picture is not comprehensively naturalistic. It is commonplace to note the unnaturalism of the gold ground, of haloes, angels and disparities of scale. In retaining these elements and characteristics, Masaccio's picture is iconographically akin to the *Ognissanti* and *Rucellai Madonnas*. Whereas in the latter works, rational and geometrical collisions accompanied the application of mobile perspectives, the static perspective of the *Pisa Madonna and Child*—so logical in itself—generates collisions no less serious, but now for the cohesion of the image in terms of the rules of hierarchy and devotion that had prevailed in earlier devotional art.

The one-point perspective system was taken up enthusiastically by painters soon after its invention. However, as we stated at the very beginning, its logic ran counter to that of devotion. Whilst it attempted to represent the laws of nature, the verities of religion continued to demand exposition, and in quite different terms. Observable in altar pictures of the period are the stratagems of painters to obey both codes, and of course obeying neither.¹²

Fra Angelico (c. 1387-1455) seems to have followed the specific instruction of *De pictura* in the *San Marco Altarpiece* of about 1438 in the Museo di San Marco, Florence (see Figure 11 on the next page). The carpet does

¹¹For an extended discussion of the Pisa Polyptych and of Masaccio's understanding of cast shadow, see [7].

¹²A third function of the altarpiece may be remarked here but must go undiscussed. One of its functions, as we have stated, is to lay out hierarchy and dogmatics. Another, opened up with picture space, is its empathetic plausibility. In its educative action, it now addresses the heart. Its third function is in relation to the altar-table and the ritual enacted there. It invokes faith. It is important to have a notion of the drama to which the altarpiece is, with the faithful, witness. An essential undoing of the laws of nature happens daily on the altar. The bread and the wine of the Eucharist are transubstantiated. A miraculous alchemy takes place, and they become literally the substance of the body and blood of Christ. In other words, the altarpiece is party to the supernatural. We get a good sense of this remarkable and—to the positivist—confounding state of things from Charles Trinkaus's account of three 15th-century sermons on the Eucharist in [9, pages 633–647].



Figure 11: Fra Angelico (c. 1400–55), *San Marco Altarpiece*, c. 1438, Museo di San Marco, Florence. Image [Public domain], via Wikimedia Commons (http://en.wikipedia.org/wiki/San_Marco_Altarpiece, accessed December 23, 2014).

service as Alberti's pavement, supplying a measure for nearly all that has spacial extension within the picture. The orthogonals point to the vanishing point, as-it-were to their goal. The significance of the point in picture space matches up with the point of iconographic focus and the point on the picture plane marking the apex of the hieratic scheme, the top of the triangle. The point has a triple role. However, it is not easy to make them coincide simultaneously in the terms of religious dogma and naturalism. The basic problem of the one-point perspective system is that the vanishing point is most remote in the picture space from the viewer; and size diminishes with distance. The Madonna of the San Marco Altarpiece turns out to be smaller on the picture plane than the attendant saints.

Such a breach of decorum is impermissible. Obedience to decorum is of greater force than that due to a scientific and naturalistic view of the world. Therefore, Fra Angelico breaks nature's laws. If she would stand up, she would fill the niche. SS. Lawrence and Peter Martyr, the foreground standing figures, are on a diminutive scale by comparison. Fra Angelico has done two things to guarantee that the Madonna occupies her position of devotional pre-eminence: he has inserted a pedestal of steps below her and he has built her on a giant scale. In order that the anomaly draw attention as little as possible, he has made the Madonna and Child's immediate companions the angels, who are allowed to be on a different and indeed indeterminate scale, and he groups the attendant saints—conceived to be on a standard human scale—well off to the sides and below. The Madonna and Child are aloof, and contributing to the effect is the very considerable space that separates them from the pious observer. Yet, even with their scale modified, the figures of the Madonna and the Child occupy a very small proportion of the picture plane. For a devotional painting this could have seemed a shortcoming.

Domenico Veneziano (c. 1410–61) gives the Madonna and Child a much larger proportion of the painted surface of the *Magnoli Altarpiece* (c. 1444) in the Uffizi (see Figure 12 on the next page). Before a loggia in an airy garden, the Madonna and Child are enthroned and accompanied by four saints, similarly of large scale in relation to the area of the panel. A certain psychological intimacy with the cast of this *Sacra Conversazione* seems possible.

The first impression is that this is an earthbound scene and one governed satisfactorily by the laws of nature. Space and light—both forms of geometry—conduct themselves with what looks like impeccable logic. The foreshortening of the pavement in the foreground indicates a relatively long viewing distance. Eventually, we register that the columns of the loggia must be very distant and very much taller than at first they appeared—then, we registered their role in creating a triptych format for the altarpiece. This is a cunning move on Domenico's part, for the honour of naturalism and devotional tradition seem both to be satisfied.

But the simultaneous functioning of the columns in measured perspective space at the picture plane (the viewer has no way of locating the spandrels of the arches in picture space except by tracing their location down though the columns and across the pavement) starts to seem remarkably fortuitous. Coincidence tends to question the happenstance that is the essence of naturalism. In fact, forms of knowledge both visual and non-visual are themes

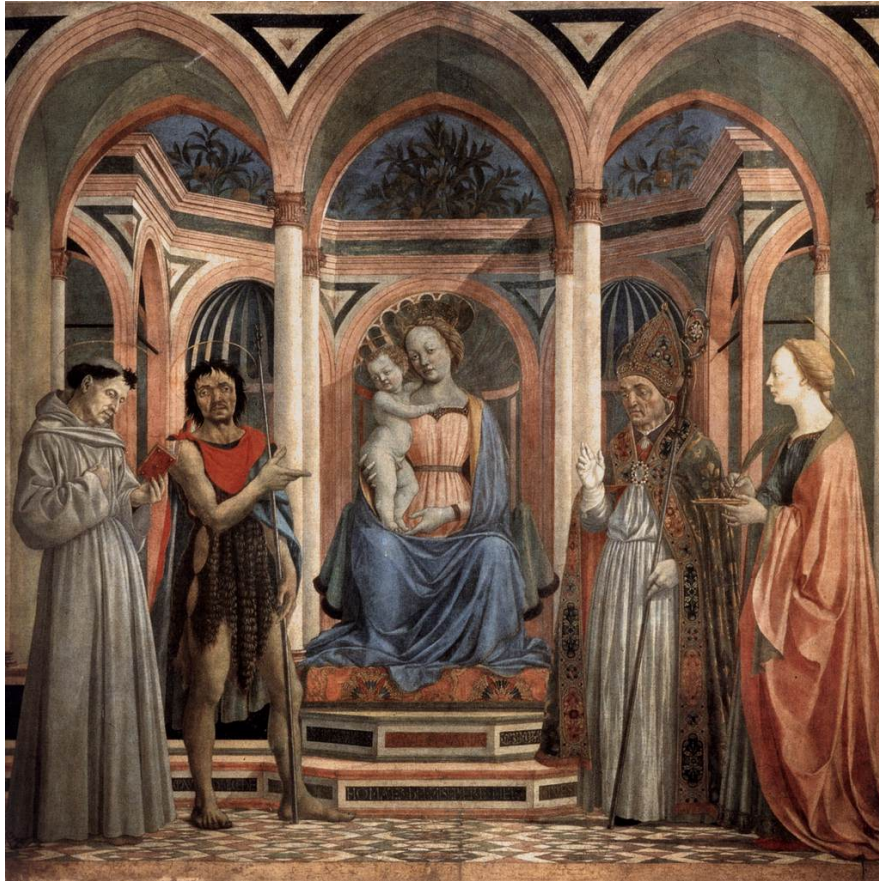


Figure 12: Domenico Veneziano (c. 1410–61), *Magnoli Altarpiece*, c. 1446, Uffizi, Florence. Image [Public domain], via Wikimedia Commons (http://en.wikipedia.org/wiki/Domenico_Veneziano, accessed December 23, 2014).

of meditation before the picture, and the incomplete and provisional nature of rational vision where religious verities are concerned is brought clearly to mind. Contradiction cannot be far off.

This piece is often called the *St. Lucy Altarpiece*. The martyr saint put out her own eyes so that their beauty would not be alluring to the suitor whom she rejected in order to preserve her chastity. She stands to right bearing her attributes—a palm of victory and her eyes on a small platter. So, sightless, she yet sees the Madonna and Child. It is by some other faculty. Her counterpart is the Franciscan saint to left. He reads his little book and, in thought, is in company with the Madonna and Child. St. John the Baptist

looks as far as the viewing distance of the perspective of the picture. He directs the viewer's eye to the Madonna who returns his gaze. None of the four saints look at the Madonna and Child. The Bishop-saint Zenobius looks downward and blesses either an invisible celebrant of the mass kneeling at the foreground of the pavement, his eye aligned with the vanishing point of the construction below the Madonna's knee-level, or else the elements of the Eucharist on the altar itself. These actions, with the exception of St John's, question the primacy of vision as a means of knowledge of things divine. In one sense, naturalism is not of supreme authority. There remain facts that vision would traduce. So, again, the Madonna may be invited to stand up; and she dwarves the other figures as the *Ognissanti Madonna* had done. However, there is another sense in which naturalism has prevailed. It has been possible to distinguish the mental actions of the saints because it has first been possible, as with the example of the *Ideal City View*, to wander the space in imagination and adopt any perspective on any of the rationally-composed objects within the rationally drafted space.

There had been collisions between geometries where the mobile perspectives of space, plane and line had been applied. The perspective of point ordered the scene.¹³ However, conflict was not resolved; it was now between nature, cast in the frame of reason, and Christian dogmatics which had been framed in the essentially planar geometry of hierarchy, specifically the triangle. The painter of the altarpiece had two masters. But the contradictory situation brought from him feats of great ingenuity and creativity.

The triangle continued to inhabit the naturalistic scene, as orthogonals pass off to the vanishing point. It is tempting to question whether devotional painting would ever have had use for the one-point perspective system if it had not involved scribing a triangle on the prepared panel before going on to paint the scene. But a perfect coincidence of the triangle on the picture plane and in the picture space brought problems of scaling, and the coincidence had to be to the detriment of naturalism.

¹³The one-point system does not produce optical illusion. That is because it does not originate in a definition of vision, but in the establishment of the projection- or construction-point. It happens that the eye can take up a position at that point. The system actually produces effects that vision contradicts. For example, a close viewing distance causes the floor-plane to seem to curved downward at the viewer's feet. Ellipses become distorted the further they are from the vertical plane of vision.

Piero della Francesca (1410/20–92), in the *Brera Altarpiece* (c. 1474), continued the practice of his one-time teacher, Domenico Veneziano, and sought to obscure the geometry by making the organic content of the image—the figures—dominate the picture plane (see Figure 13 below). Saints, angels and Madonna are arranged in isocephalic manner. They also form a very compact and therefore intimate group. By both means, the Madonna’s aloofness is reduced. There is scarcely space between the edges of the panel and the figures of John the Baptist to left and St John to right to register the extent of pavement behind the holy group and recognize that, as in the *Magnoli Altarpiece*, the architecture, impeccably rendered according to the one-point system is grand and distant, while it frames the figures snugly at the picture-plane.

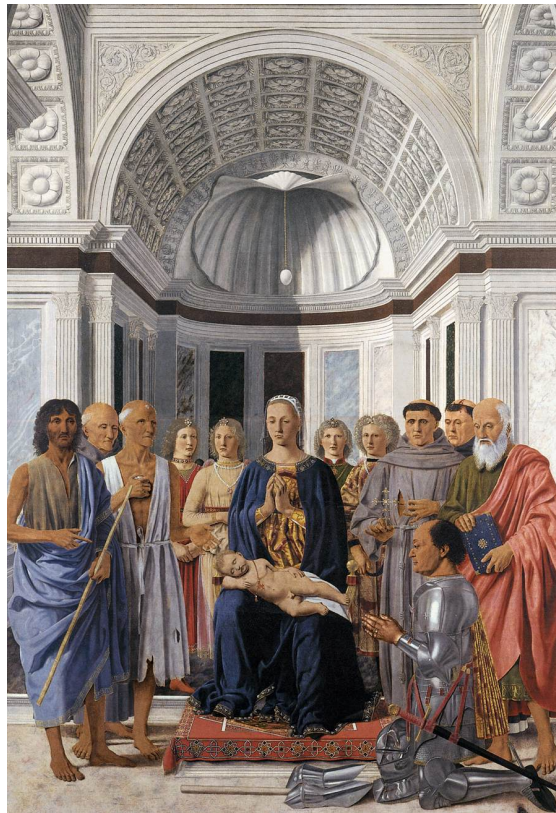


Figure 13: Piero della Francesca (1416–92), *Brera Altarpiece*, c. 1472, Museo del Brera, Milan. Image from Web Gallery of Art, at http://www.wga.hu/html_m/p/piero/3/12monte1.html, accessed on December 23, 2014.

The conflict of surface and space is exquisitely suspended in Piero's altarpiece. The logic of surface does a remarkable thing: it directs the viewer's empathetic reading of the scene. There is also a lower isocephalic level. It joins the kneeling donor, Federigo da Montefeltro, ruler of Urbino, and the sleeping Child. Whilst Federigo, to the right, sees the Child, he also has other faculties with which to see, like Domenico's St Lucy, the Franciscan saint and Saint Zanobius. With his inner eye, he sees the cross that is held by St Francis who is immediately behind him. And he sees the stigmata that St Francis exposes. A rhyme must be striking Federigo, for the Child bears a necklace, and a branch of coral lies against his right side at the corresponding place, like petrified blood. From the argument that Federigo is thinking about the Cross and what it implies follows the obligation to take note of the book held so prominently behind his head by St John. It is easy to overlook as a specific prompt to pious speculation. However, an attentive reading of the picture at this point proposes that Federigo sees in another sense too: he sees the Word. This is the same Word, that "...the world knew not" [John: I.10] and which underwent the Passion. In full armour, Federigo promises to take up the Cross for Christ.

There is a considerable accumulation of religious meaning in this area of the picture plane and in this shallow envelope of picture space. And there is a stark absence of it at the corresponding part of the picture to the left. Federigo and the Madonna fill out a right-angled triangle. The expectations and the demands of symmetry scribe insistently in the observer's mind another one on the other side of the axis. In other words, and remarkably, Piero has been able to indicate an absence. Perhaps Saint Zenobius put him partly in mind of the possibility; but mostly it has been the persistence of the ancient schema of the triangle. The donor who is so eloquently not there is the namesake of the Baptist and the one to whom he points, Battista Sforza, Federigo's wife—who died shortly after giving birth to a son. This lower isocephalic level then has the invisible Battista to left. Federigo's vision, literal and imaginative, is of a mother and a child, humanity and divinity interchangeable in his mind.

The observer of the altarpiece who knew the first famous verses of the Gospel of St John might have pondered the thought that a particular verse was ringing in Federigo's mind as he contemplates Battista and Guidobaldo overlaying and underlying the Madonna and Child: "All things were made

by him and without him was not any thing made that was made.” [I.3] The altarpiece would include the character of an *ex voto*.

The clarity of the space, the consistency of the light and the classical rationality of the architecture constitute a representation emphatically of the world in harmony with nature’s laws. The law of gravity has total authority, so that the angels themselves are earthbound. All the while though, the uneluctable facts that the religion insists upon must be respected. Vision is not just a matter of optics. Mobility is not entirely replaced by the stilled. The Madonna, invited to stand up, has not diminished much in stature since Duccio and Giotto. With, however, the invention of the one-point perspective system, a new kind of mobility was also possible. Within the rationally constructed picture space any perspective was imaginable, including that of the participants in the action. What they see in the literal sense being accessible, what they see in those other senses—understand, conceive, remember, witness, foretell—might, through the faculty of empathy, be known.

Born in 1483 in the small city of Urbino, where Federigo da Montefeltro’s altarpiece was to be seen, was an artist who made paint conjure up, for the viewer, people of the fullest mental life. To be able to impute thought and feeling to people made of no more than paint on a surface is extraordinary. Their precondition is picture space. It can be proposed that Piero’s work—the product of a history and a culture set upon the task of reconciling eternal meaning and the fact of life lived and concluded in time—served as a lesson to Raphael.

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